Distributed-Mode Loudspeaker
Radiation Simulation

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A radiation model of the Distributed Mode Loudspeaker (DML) is investigated and compared to measurements. The approach makes use of the bending wave eigen-functions and Fourier transformation to describe the acoustic coupling. The model is implemented into a lumped element simulator, which helps to display the complete system response including exciter and other components.

Introduction

The Distributed Mode Loudspeaker is an electro-acoustic transducer based on the direct radiation of modal bending waves [1-3]. The DML mechanics are described through partial differential equations and with the help of boundary conditions for the structure and fluid. The fluid boundary is the interface between the vibrating panel surface and the pressure of the reacting medium in which the DML is embedded.

The radiation of the DML is governed by the wave-equation, which is again a partial differential equation. Acoustic boundary conditions determine the solution. The acoustic field must match the vibrating surface of the panel and other surfaces in the radiation space. In this paper we investigate the far-field radiation pattern in free space.

Bending Wave Radiation

Assuming linearity, the radiating bending wave can be expressed as the superposition of plane transversal velocity waves. A single plane wave can be regarded as a sub-module in order to describe complex field patterns. Therefore let us first investigate the radiation of these sub-modules. In one dimension and normal to the panel surface a single plane bending wave may have the form (time factor exp(jωt) suppressed) [4]:

\[ v(x) = A \cdot e^{-j\kappa x} \]

(1)

with
\[ k_b = \sqrt{\frac{\omega^2 \cdot \mu}{B}} \]  \hspace{1cm} \text{Bending wave number [m}^{-1}\text{]} \\
B \hspace{1cm} \text{Bending stiffness [Nm]} \\
\mu \hspace{1cm} \text{Mass surface density [kg/m}^2\text{]} \\
A \hspace{1cm} \text{Arbitrarily constant}

The generated sound pressure satisfies the wave equation and the boundary conditions of continuity at the air-panel interface. A plane transversal surface wave can only radiate if it matches to the trace of a longitudinal acoustic wave. The trace of the acoustical wave is the projection onto the panel surface.

Figure 1 displays a section through the panel where a bending wave travels in the x direction. Above, the grid displays the sound pressure variation. The left picture of Figure 1 demonstrates the situation when the transversal bending wavelength is smaller than the wavelength in air. The air wave cannot project a trace on the bending wave. There is only a fluctuation very near to the surface exponentially decaying in the z-direction. Hence no real sound power is radiated.

The right picture displays the sound field above coincidence, where the bending wavelength is longer than the wavelength in air. The direction of radiation is called coincidence angle \( \vartheta_c \) and is related to the speed of sound in air, \( c_o \):

\[ \sin(\vartheta_c) = \frac{\lambda_o}{\lambda_b} = \frac{c_o}{\sqrt{\omega_o}} \cdot \frac{\mu}{\sqrt{B}} \]  \hspace{1cm} (2)

The frequency where the wavelength is the same for the bending wave and for the sound wave in air is called coincidence frequency, \( f_c \):

\[ f_c = \frac{c_o^2}{2 \cdot \pi} \cdot \frac{\mu}{\sqrt{B}} \]  \hspace{1cm} (3)
Far-Field Response

The far-field response of any vibrating area in an infinite baffle is proportional to the velocity wavenumber spectrum [5].

\[ p(R,k_o,\theta,\phi) = e^{-jk_o R} \cdot \bar{v}(k_o,\theta,\phi) \mid_{R \to \infty} \]

with

\[ p(R,k_o,\theta,\phi) \] Sound pressure at frequency \( f = k_o \cdot c_o / 2\pi \),

\[ \theta, \phi \] listening angle \( \theta, \phi \) and distance R.

\[ Z_o \] Specific plane wave impedance \( Z_o = \rho_o c_o \)

\[ \rho_o, c_o \] Density and sound velocity of air

\[ \bar{v}(k_o,\theta,\phi) \] Spatial Fourier-transform of panel velocity.

Arguments mapped to listening angles \( \theta \) and \( \phi \).

The velocity wavenumber spectrum, \( \bar{v}(k_x,k_y) \), is the two dimensional Fourier transform of the radiating velocity field due to a bending wave in the xy-plane: \( \bar{v}(k_x,k_y) = \int v(x,y) \cdot e^{j(k_x x + k_y y)} \, dx \, dy \). Thus the radiating velocity profile of the plate is described with the help of plane waves.

From this spatial spectrum only those wavenumber components whose k-factor is smaller than the wavenumber, \( k_o \), radiate into the far-field, as explained above. The wavenumbers are related to the listening angles \( \theta \) and \( \phi \) by (Figure 2)

\[ k_{ox} = k_o \cdot \sin \theta \cdot \cos \phi \quad \text{and} \quad k_{oy} = k_o \cdot \sin \theta \cdot \sin \phi \]

On axis response gives \( k_{ox} = 0 \) and \( k_{oy} = 0 \). The maximum range which can be covered is given by the radius \( k_o \) as illustrated in the right diagram of Figure 2.
Aperture Function

Let us call any formulation for the mechanical velocity profile normal to the panel, \( v_{o(x,y)} \). On the baffle, outside the vibrating plate, the normal component of velocity is zero, since we assume a reflecting, rigid and infinitely large baffle. Thus any formulation of the acoustic velocity profile is strictly limited to the vibrating surface. The acoustically active velocity function is therefore: \( v_{(k_x,k_y)} = v_{o(x,y)}A_{(x,y)} \). We call \( A_{(x,y)} \) the aperture function. \( A_{(x,y)} = 1 \) for \( x,y \) inside the radiating area and zero elsewhere as illustrated in Figure 3. The spatial Fourier transform of \( v_{(k_x,k_y)} = v_{o(x,y)}A_{(x,y)} \) is therefore the convolution of \( \tilde{v}_{o(k_x,k_y)} \) with \( \tilde{A}_{(k_x,k_y)} \), which is the spatial Fourier transform of \( A_{(x,y)} \):

\[
\tilde{v}_{(k_x,k_y)} = \tilde{v}_{o(k_x,k_y)} \ast \tilde{A}_{(k_x,k_y)}
\]  

This result is important since it enables the radiation of subsonic plane bending waves. As soon as we apply any limits to the velocity field the acoustic field is distorted, as illustrated in Figure 3. The cancellation process below coincidence is now leaky and some of the energy can be transduced into the far-field. Above coincidence the directivity is smoothed by the aperture spectrum \( \tilde{A}_{(k_x,k_y)} \) and radiation occurs in all directions.

Figure 4 displays the near field of the sound pressure, which is created by a single plane bending wave according to equation (1) but now limited to the size of a panel. The plots display the linearly scaled sound pressure contours. The distortion of the sound field contours is clearly visible. The parameters used are the same as for the following examples of the large panel. At the bottom of the diagrams the finite vibrating panel area is symbolized with the help of an aperture covering an infinite plane bending wave.

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Examples

In order to further demonstrate the principles of bending wave radiation, let us investigate an example of the radiation of an infinitely large panel with and without an aperture. This intermediate step has the advantage of enabling the study of the bending wave radiation of a finite aperture without the involved details of modal vibration of the finite panel. The material properties of the sample panel shall be such that the bending stiffness is $B = 10 \text{ Nm}$ and the mass surface density is $\mu = 1.015 \text{ kg/m}^3$. With the above values for $B$ and $\mu$ the coincidence frequency is $f_c = 6 \text{ kHz}$ corresponding to $k_c = 109.6 \text{ m}^{-1}$.

On the left hand side of the following series of diagrams the wavenumber spectrum is displayed. For the sake of clarity we display the properties for one dimension only. The abscissa is therefore the wavenumber $k_x$ in the $x$-direction. The ordinate is the sound pressure arbitrarily scaled since the actual level is not of interest here. On the right hand side the associated polar plot according to equation (5) is displayed. The ordinate of the polar plot is arbitrarily log scaled.

1. No aperture - below coincidence

In this first example let us assume a free bending wave $\cos(k_b x)$ of $f = 4 \text{ kHz}$, which is below coincidence. $k_b$ is marked in Figure 5 with the help of two arrows, because the Fourier transform of a single plane wave is a Dirac function. No radiation can occur, which an empty directivity plot on the right hand side symbolizes (see also Figure 1, right diagram). According to the mapping of (5) the listening angle covers the range of $k_x = -73 \text{ m}^{-1} ... +73 \text{ m}^{-1}$ while the angle $\theta$ runs from $-90^\circ$ ... $+90^\circ$. The range is marked with the help of dotted lines. The given frequency, $f$, of our free bending wave corresponds to a wave number of $k_b = 89.5 \text{ m}^{-1}$, which is outside of the listening angle range where the limit is given by $k_x = 73 \text{ m}^{-1}$.

Figure 5 No aperture - below coincidence, $f = 4 \text{ kHz}$
2. **No aperture - above coincidence**

![Image of a graph showing no aperture above coincidence, f = 8 kHz](image1)

In this example the free bending wave \( \cos(k_b \cdot x) \) is above coincidence, at \( f = 8 \text{ kHz} \). The listening angle range now includes the two Dirac functions (Figure 6). Hence a strong radiation occurs in a direction where the trace of the acoustic wave matches the bending wave of the panel (see also left diagram of Figure 1).

3. **Large aperture - below coincidence**

![Image of a graph showing large aperture below coincidence, f = 4 kHz](image2)

As illustrated in Figure 3 and Figure 4 the infinite panel is now covered by an opaque screen containing a slit of width \( A_x = 0.5 \text{ m} \). We assume a plane bending wave \( \cos(k_b \cdot x) \) travelling in the \( x \)-direction. The origin is placed in the center of the aperture. In spite of driving below coincidence a far-field radiation is possible. The reason is the distorted acoustic field as sketched in Figure 3. At some point, generated sound pressure cannot be totally cancelled by an inverted pressure at some other point. The residual forms the radiated sound power. The power is related to the grayed zone under the curve in Figure 7, i.e. exactly the range, which can be covered by the listening angle. On the right hand side of Figure 7 this range is plotted again in form of the directivity plot. Compared to Figure 4 it can be seen how the far-field pattern evolves from the aperture.
4. Large aperture - above coincidence

![Figure 8 Large aperture - above coincidence; \( A_x = 0.5 \text{ m}, f = 8 \text{ kHz} \)](image)

We have the same situation as in the previous example but our bending wave corresponds now to a frequency of \( f = 8 \text{ kHz} \), i.e. well above coincidence of \( f_c = 6 \text{ kHz} \). The 'coincidence beams' are within the range, which the listening angle is able to cover (grayed zone in Figure 8). The Dirac function of example 2 is now convolved with the aperture spectrum. The polar plot displays the strongly emphasized radiation at the coincidence angle \( \theta_c \). The right diagram of Figure 4 displays the corresponding near-field at the aperture.

Exactly at coincidence \((f = f_c = 6 \text{ kHz})\) the range of the listening angle would end at the peaks of Figure 8, i.e. the coincidence angle is in this particular case \( \theta_c = \pm 90^\circ \).

5. Small aperture - below coincidence

![Figure 9 Small aperture - below coincidence; \( A_x = 0.25 \text{ m}, f = 3 \text{ kHz} \)](image)

This example demonstrates the effect of the aperture-size below coincidence. The situation is the same as in example 3 but the width of the slit is halved \((A_x = 0.25 \text{ m})\). The smaller aperture causes a stronger distortion of the acoustic field. Hence a stronger radiation can be observed as demonstrated in Figure 9.
6. Small aperture - above coincidence

![Figure 10 - Small aperture - above coincidence; A_x = 0.25 m, f = 8 kHz](image)

Above coincidence the effect of halving the slit width of the aperture (A_x = 0.25 m) increases the width of the coincidence beams as shown in Figure 10.

By comparing the results of the examples 1 to 6 it is obvious that the aperture size controls the directivity. Below coincidence a large aperture (in comparison to the wavelength) diminishes the acoustic coupling. Above coincidence the radiated energy is focused into a certain direction called coincidence angle. Below coincidence a small aperture increases the radiation. Above coincidence the radiation in the neighbourhood of the coincidence angle is broadened.

In two dimensions similar results are obtained because a plane wave travelling in any direction can always be separated into the components relating to the x and y directions.

**The Finite Panel**

So far we have investigated the interaction of a single plane bending wave radiating through an aperture. Hence there are no reflecting bending waves and no boundary conditions to be matched. In a finite plate on the other hand, the bending wave field has to satisfy boundary conditions. For example for a finite free vibrating panel, there are no external forces and moments applied at its edges. Of course, any panel of finite size comes automatically with an 'in-built' aperture function A_{x,y}, the size of which is usually the same as that of the vibrating area of the panel. The velocity profile of a finite panel can be described with the help of a series expansion in a linear independent function system. Well-known examples are the Fourier or Hankel series where the function system consists of sin-functions or Bessel-functions, respectively. For rectangular panels an approach, which uses so-called beam-functions, has certain advantages. The beam-functions are themselves a series of sin-functions and those properties are closely related to plate mechanics. Without going into details here, we can thus always write our panel velocity profile in the form
\[ v_o(x,y) = \sum v_i \cdot \phi_i(x,y) \]  \hspace{1cm} (7)

with

\[
v_o(x,y) \quad \text{Bending wave velocity at point } x, y \\
i \quad \text{Mode number} \\
v_i \quad \text{i-th mode coefficient} \\
\phi_i(x,y) \quad \text{i-th mode shape or eigen function}
\]

The velocity profile \( v_o(x,y) \) has to be a solution of the governing plate differential equation and has to match the boundary conditions at the edge of the panel as displayed in Figure 3. Both criteria determine the choice of eigen-functions \( \phi_i(x,y) \) and their corresponding eigen-values for each mode, \( i \). Together with these, the mode coefficients \( v_i \) are formed through the location of driving points, material parameters and radiation impedance. For the purpose of this paper the involved details of \( v_i \) and \( \phi_i(x,y) \) are not necessary.

As mentioned above the acoustical active velocity is \( v(x,y) = v_o(x,y) \cdot A(x,y) \). Since the mode coefficients \( v_i \) are independent of \( x \) and \( y \) the only step we have to do is to apply the spatial Fourier transformation to the mode shape functions \( \phi_i(x,y) \), i.e.

\[
\bar{v}_{(h_x,k_y)} = \sum v_i \cdot \bar{\phi}_{i(h_x,k_y)} \ast \bar{A}_{(h_x,k_y)} \]  \hspace{1cm} (8)

Thus inserting (8) into the far-field Rayleigh formula (4), the far-field response of the finite panel in a baffle is the convolution of the Fourier spectrum of eigen-functions with the spectrum of the aperture.

**Examples**

As already mentioned we are in some sense free to choose an eigen-function system. The proper choice depends mainly on numerical reasons and on the way the input parameters are given. As an example we implement the in-vacuum beam-model [6]. Since the eigen values of this model do not take into account the radiation impedance there is a coupling between the mode coefficients \( v_i \), which has to be calculated separately. Further the aperture function \( A(x,y) \) is not included in the beam eigen-functions and must be applied explicitly as given by equation (8).

The following comparisons between simulation and measurement display the response of (8) in principle. These should be regarded as an intermediate report of development of the DML radiation model. The beam-mode-model for panels is implemented in a lumped element network simulator [7]. Some parts are missing in the following simulation, for example the mode model for the driving force and the diffraction. The electro-mechanical part is the same as described in [8], i.e. the driving force is acting on the impedance of an infinite panel, which is mainly a constant. In reality the modal impedance fluctuates strongly around this constant. Thus the driving power for each mode fluctuates accordingly. Since the sum of all modes controls the directivity, the frequency response of the input has an effect on the
directivity. Further the current model does not include diffraction. In order to take into account some aspects of two-sided radiation (no in-plane pressure) a model for an absorbing baffle is used.

The points on the polar plots are measured in 5° steps in a reflection free environment. The measured panels are unbaffled and the response may include diffraction from the rearward side. The absolute scaling of the diagrams is ignored but the range is maintained and is 50 dB.

The used panels are not particularly selected to be good ones for loudspeaker design. The selection ensured a light, stiff and low loss material.

1. **Small panel**

Panel parameter

Size: $L_x = 260\,\text{mm}, L_y = 230\,\text{mm}$ (x - horizontal)

Material: Sandwich, plastic skin on foam, isotropic, $B = 1.3\,\text{Nm}, \mu = 0.72\,\text{kg/m}^2$

Exciter: Single, at $x = 150\,\text{mm}, y = 100\,\text{mm}$, voice coil diameter $dVc = 25.5\,\text{mm}$

Edges: Free

Coincidence: $f_c = 14\,\text{kHz}$

The directivity plots of this panel are displayed in Figure 11. Clearly visible is the slightly asymmetric radiation pattern because the exciter is mounted off-center. Below coincidence the directivity is almost omni-directional. At higher frequencies coincidence lobes start to form the typical radiation pattern of DMLs at and above coincidence. The plot at 2 kHz is affected by interference between the rearward radiated sound with the direct one. The mean pathlength between the center of the rearward side to the front is half a wavelength at 2.8 kHz, where the strongest cancellation occurs on-axis.
Figure 11 Directivity: small panel, simulation - measurement
2. **Large panel**

Panel parameter

<table>
<thead>
<tr>
<th>Size:</th>
<th>( L_x = 600 \text{ mm}, L_y = 600 \text{ mm} ) (x - horizontal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material:</td>
<td>Sandwich, plastic skin on honeycomb, orthotropic, mean ( B = 7 \text{ Nm}, \mu = 1.2 \text{ kg/m}^2 )</td>
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<tr>
<td>Exciter:</td>
<td>Single, at ( x = 265 \text{ mm}, y = 330 \text{ mm} ), voice coil diameter ( d_{Vc} = 25.5 \text{ mm} )</td>
</tr>
<tr>
<td>Edges:</td>
<td>Free</td>
</tr>
<tr>
<td>Coincidence:</td>
<td>( f_c = 7.8 \text{ kHz} )</td>
</tr>
</tbody>
</table>

Figure 12 displays the directivity plots of the larger panel. The lower coincidence frequency together with the relative large size leads to stronger side radiation, which already starts above 2 kHz. Clearly visible are the 'coincidence beams' at high frequencies. Again interference caused by pathlength difference between rear and front side causes the indentation at the 2 kHz and 5 kHz measurement plots.
Figure 12 Directivity: large panel, simulation - measurement
Conclusion

The principles of bending wave radiation are explained step by step. Starting with the radiation of a single one dimensional plane wave below and above coincidence. The effect of an opaque screen with an aperture on the radiation is demonstrated. The aperture effect is used for explanation of the general baffled far-field radiation formula. Several examples are given. A simulation model for a finite panel is demonstrated and discussed in comparison with measurements.

Further research is focusing on a complete modal transmission system for the distributed mode loudspeaker, i.e. also including the mechanical part. An important area of investigation is what effect diffraction has on a radiating panel.

Acknowledgement

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References


[8] Panzer, J. W.; Harris, N.; Distributed Mode Loudspeaker Simulation Model; AES 104th Convention May 1998